

Supercooling in viscous hydrodynamics for QCD phase transition

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Abstract

First order quark hadron phase transition is considered in scaling hydrodynamics including the Csernai-Kapusta model of nucleation of hadronic bubbles leading to supercooling. The effect of viscosity on the entropy production is studied in ideal as well as in viscous hydrodynamics with and without supercooling. It is found that the excess entropy produced due to supercooling depends weakly on viscosity.

1 Introduction

The ultrarelativistic heavy ion collisions provide a means to create a new state of matter at high temperature and density as the quantum chromodynamics (QCD) predicts a phase transition from normal hadronic matter to quark gluon plasma (QGP), a state of unconfined quarks and gluons. In recent years a considerable amount of theoretical and experimental activity is going on in this field. After the collision of two Lorentz contracted nuclei, QGP is assumed to be formed and equilibrated in a very small time of the order of ~ 1 fm. In the usual description of the evolution of the plasma, it cools by expanding hydrodynamically till the critical temperature T_c is reached at which a transition from QGP to normal hadronic matter takes place. The temperature remains fixed at T_c until hadronization gets completed. This hadronization scenario corresponds to the ideal Maxwell construction. However, in reality hadronization does not begin at $T = T_c$ due to the large nucleation barrier. Recently Csernai and Kapusta have proposed a model for nucleation for the relativistic first order phase transition [1]. Supercooling, through nucleation of hadronic bubbles in QGP has been studied by several authors [1, 2, 3, 4] using the Csernai-Kapusta model of nucleation. A general outcome of these studies is that the plasma will cool according to the law $T(\tau) = T_0(\tau_0/\tau)^{1/3}$ till T_c . The matter continues to cool below T_c until the temperature goes down to about $\sim .8 T_c$, where bubble formation and growth becomes sufficient to reheat the system due to the release of latent heat. Compared to the idealized Maxwell construction the supercooling delays the transition and leads to an extra entropy production as the nucleation process allows dissipation around the hadronic bubbles. The dynamical prefactor [1] in the nucleation

¹Ref: Phy. Rev. C **59**, 914 (1999)

rate includes quark viscosity coefficients which bring dissipative effect in the medium. In fact, the nucleation rate is limited by the ability of the dissipative processes to carry latent heat away from the bubbles's surface. However, for the dynamical evolution of the plasma the ideal hydrodynamics is used. This is not consistent as the viscosity dependent terms in hydrodynamics would also contribute to the entropy production. Therefore, in this work we study the supercooling in the viscous hydrodynamics. The role of viscosity on supercooling and entropy production has been investigated in detail.

2 Csernai-Kapusta model of nucleation

The nucleation model computes the probability that a bubble of the hadronic matter appears in a system, initially in QGP phase near the critical temperature. Further in the model a small baryon chemical potential is assumed. The bubble formation is assumed to take place in a homogeneous QGP phase consisting of u and d quarks and gluons, ignoring the role of inhomogenities such as strange quarks. It is further assumed that there is not substantial supercooling. Langer's theory of nucleation gives the nucleation rate per unit volume at temperature T as

$$I = \frac{\kappa}{2\pi} \frac{\Omega_0}{V} e^{-\Delta F_*/T}, \quad (1)$$

where ΔF_* is the change in the free energy of the system due to the formation of a critical hadronic droplet, Ω_0 is a statistical prefactor which measures the available phase volume and V is the volume of the system. The dynamical prefactor, κ determines the exponential growth rate of critical droplets which are perturbed from their equilibrium radius R_* . The coarse-grained effective field theory approximation to QCD is utilised to obtain κ and Ω_0 . The factor Ω_0 is obtained as

$$\frac{\Omega_0}{V} = \frac{2}{3} \left(\frac{\sigma}{3T} \right)^{3/2} \left(\frac{R_*}{\xi_q} \right)^4, \quad (2)$$

where σ is the surface free energy and is determined as 50 MeV/fm² by lattice gauge theory simulations without dynamical quarks. The correlation length ξ_q is estimated as 0.7 fm. The dynamical prefactor κ is obtained as

$$\kappa = \frac{4\sigma(4/3\eta_q + \zeta_q)}{(\Delta\omega)^2 R_*^3}, \quad (3)$$

where $\Delta\omega$ is the difference in the enthalpy densities of the two phases. η and ζ are respectively, the shear and bulk viscosity coefficients. Inserting Ω_0 from Eq. (2) and κ from Eq. (3) in Eq. (1) we get the nucleation rate per unit volume as

$$I = \frac{4}{\pi} \left(\frac{\sigma}{3T} \right)^{3/2} \frac{\sigma(4\eta_q/3 + \zeta_q)R_*}{\xi_q^4(\Delta\omega)^2} e^{-\Delta F_*/T}. \quad (4)$$

The critical radius R_* is given by the Laplace formula as

$$R_*(T) = \frac{2\sigma}{p_h(T) - p_q(T)}, \quad (5)$$

where $p_{q/h}$ is the pressure of the quark/hadron phase at temperature T and the ΔF_* as

$$\Delta F_* = \frac{4\pi}{3} \sigma R_*^2. \quad (6)$$

From the nucleation rate Eq. (4), one can calculate the fraction of volume $h(\tau)$ which has been converted from QCD plasma to hadronic gas at proper time τ . If the system cools to T_c at time τ_c , then at some later time τ the fraction h of space which has been converted to hadronic gas is

$$h(\tau) = \int_{\tau_c}^{\tau} d\tau' I(T(\tau')) [1 - h(\tau')] V(\tau', \tau). \quad (7)$$

Here $V(\tau', \tau)$ is the volume of a bubble at time τ which had been nucleated at an earlier time τ' ; this takes into account the bubble growth. The factor $1 - h(\tau')$ accounts for the fact that new bubbles can only be nucleated in the fraction of space not already occupied by the hadronic gas. The model for bubble growth is simply taken as [5]

$$V(\tau', \tau) = \frac{4\pi}{3} \left(R_*(T(\tau')) + \int_{\tau'}^{\tau} d\tau'' v(T(\tau'')) \right)^3, \quad (8)$$

where $v(T)$ is the velocity of the bubble growth at temperature T . By definition a critical size bubble is metastable and will not grow without a perturbation. The growth of bubbles has been studied numerically with relativistic hydrodynamics by Miller and Pantano [6]. Their results are consistent with the growth law

$$v(T) = 3c[1 - T/T_c]^{3/2}. \quad (9)$$

This expression is intended to apply only when $T > \frac{2}{3}T_c$ so that the growth velocity stays below the speed of sound of a massless gas, $c/\sqrt{3}$. At the critical temperature, $R_* \rightarrow \infty$, $\Delta F_* \rightarrow \infty$ and the rate of nucleation vanishes. The system must supercool at least $\sim 5\%$ to attain a finite rate. In the evolution of the matter from QGP to hadron phase, the temperature varies with τ and its description by scaling hydrodynamics provides another equation so that $h(\tau)$ and $T(\tau)$ are determined from these equations at any proper time τ .

3 Viscous hydrodynamics

The scaling viscous hydrodynamics is discussed by Danielewicz and Gyulassy [7] and others. The form of the dissipative terms depends on the choice of the definition of what constitutes the local rest frame of the fluid. The Landau-Lifshitz definition is appropriate for describing systems with small (or zero) chemical potential. Here we give a simple derivation of the viscous hydrodynamics. In scaling hydrodynamics the expansion takes place only along the direction of collision which we chose as z axis. The proper time τ and space-time rapidity y are used in place of t and z which are defined by

$$\tau = \sqrt{t^2 - z^2} \quad (10)$$

and

$$y = \frac{1}{2} \ln \frac{t+z}{t-z}. \quad (11)$$

In the rest frame of the fluid we take a volume element as $\delta V = A_\perp \tau \delta y$, where A_\perp is the area of the element transverse to z direction. The expansion takes place with the velocity $v_z = z/\tau$, in the rest frame of the element. Due to the viscous effects in longitudinal direction the heat density per unit time is given by [8]

$$\begin{aligned} \phi &= 2\eta \left(\frac{\partial v_z}{\partial z} \right)^2 + (\zeta - 2\eta/3) \left(\frac{\partial v_z}{\partial z} \right)^2, \\ \text{or} \quad \phi &= (4\eta/3 + \xi)/\tau^2. \end{aligned} \quad (12)$$

After time increment $\Delta\tau$, the volume expands by $\Delta V = A_\perp \Delta\tau \delta y$. The amount of work done in expansion is $p\Delta V$. If the energy density at τ is ϵ and at $\tau + \Delta\tau$ is $\epsilon + \Delta\epsilon$ then the energy conservation implies

$$\begin{aligned} \epsilon \delta V &= (\epsilon + \Delta\epsilon)(\delta V + \Delta V) + p\Delta V - \phi \tau \Delta V, \\ \text{or} \quad 0 &\simeq \Delta\epsilon A_\perp \tau \delta y + (\epsilon + p - \phi \tau) A_\perp \Delta\tau \delta y, \end{aligned} \quad (13)$$

which leads to the scaling Navier-Stokes equation in the limit $\Delta\tau \rightarrow 0$

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + p}{\tau} + \frac{4\eta/3 + \zeta}{\tau^2}. \quad (14)$$

Equation (14) has been solved earlier [9] for the QGP to hadron transition with the Maxwell construction, i.e., assuming $T = T_c$ for the mixed phase.

To solve Eq. (14) we require knowledge of the equation of state and the temperature dependence of η and ζ . In this work we use the bag equation of state for QGP. The energy density, pressure and entropy densities in pure QGP and hadron phases are taken as

$$\epsilon_q(T) = 3a_q T^4 + B, \quad p_q(T) = a_q T^4 - B, \quad s_q(T) = 4a_q T^3, \quad (15)$$

$$\epsilon_h(T) = 3a_h T^4, \quad p_h(T) = a_h T^4, \quad s_h(T) = 4a_h T^3. \quad (16)$$

Here, a_q and a_h are related to the degrees of freedom operating in two phases and B is the bag pressure.

For ultrarelativistic gases, the bulk viscosity ζ is usually much smaller than the shear viscosity η [5]. Danielewicz and Gyulassy [7] give the acceptable range of η for the applicability of the Navier-Stokes equation to the expansion of the plasma as

$$2T^3 \leq \eta \leq 3T^3(\tau T). \quad (17)$$

We define $\mu = (4\eta/3 + \zeta)$ and assume the temperature dependence of viscosity coefficient as

$$\mu_q(T) = \mu_{q0} T^3, \quad (18)$$

with q being replaced by h for the hadronic phase.

Solving Eq. (14) for temperature in pure quark phase, we get [9]

$$T = T_0 \left(\frac{\tau_0}{\tau} \right)^{1/3} + \frac{\mu_{q0}}{8a_q \tau_0} \left[\left(\frac{\tau_0}{\tau} \right)^{1/3} - \frac{\tau_0}{\tau} \right] \quad (19)$$

for QGP formed at time τ_0 while in pure hadron phase

$$T = T_h \left(\frac{\tau_h}{\tau} \right)^{1/3} + \frac{\mu_{h0}}{8a_h \tau_h} \left[\left(\frac{\tau_h}{\tau} \right)^{1/3} - \frac{\tau_h}{\tau} \right], \quad (20)$$

where τ_h is the time where hadronization gets completed.

The energy density in the mixed phase at a time τ can be written in terms of hadronic fraction $h(\tau)$ as

$$\begin{aligned}\epsilon(\tau) &= \epsilon_q(T) + (\epsilon_h(T) - \epsilon_q(T))h(\tau), \\ &= 3[a_q + (a_h - a_q)h(\tau)]T^4 + B[1 - h(\tau)],\end{aligned}\quad (21)$$

while the enthalpy density is

$$\omega(\tau) = 4[a_q + (a_h - a_q)h(\tau)]T^4. \quad (22)$$

The temperature T can be deduced as

$$T(\tau) = \left(\frac{\omega(\tau)}{4[a_q + (a_h - a_q)h(\tau)]} \right)^{1/4}. \quad (23)$$

The shear and bulk viscosities are also taken to be the functions of time according to

$$\begin{aligned}\mu(\tau) &= \mu_q(T) + (\mu_h(T) - \mu_q(T))h(\tau), \\ \text{or} \quad \mu(\tau) &= [\mu_{q0} + (\mu_{h0} - \mu_{q0})h(\tau)](T(\tau))^3.\end{aligned}\quad (24)$$

For the Maxwell construction, i.e., for $T = T_c$ in the mixed phase, solution of Eq. (14) is

$$h(\tau) = (c - ab)[Ei(b/\tau) - Ei(b/\tau_c)]e^{-b/\tau}/\tau + (a + 1)[1 - \tau_c/\tau e^{(b/\tau_c - b/\tau)}]. \quad (25)$$

Here, Ei is exponential integral and

$$a = \frac{4}{3} \frac{e_h}{e_q - e_h}, \quad b = \frac{\mu_q - \mu_h}{e_q - e_h}, \quad c = \frac{\mu_h}{e_q - e_h}. \quad (26)$$

The constants a , b , and c are evaluated for $T = T_c$.

The solution given by Eq. (25) does not account for supercooling. Due to supercooling, i.e., $T \neq T_c$ in the mixed phase, the temperature is not constant and depends on τ . Equation (14) then becomes

$$\frac{d\epsilon}{d\tau} = -\frac{\omega}{\tau} + \frac{\mu_{q0} + (\mu_{h0} - \mu_{q0})h(\tau)}{[4a_q + 4(a_h - a_q)h(\tau)]^{3/4}} \frac{\omega^{3/4}}{\tau^2}. \quad (27)$$

Equation (27) and Eq. (7) are coupled equations, finally to be solved for $h(\tau)$ and $\epsilon(\tau)$ [or $T(\tau)$]. Once we get $h(\tau)$ and $T(\tau)$ we can calculate the entropy density.

Eq. (27) can also be written in terms of entropy as

$$\frac{ds}{d\tau} = -\frac{s}{\tau} + \left(\frac{p_h(T) - p_q(T)}{T} \right) \frac{dh}{dt} + \frac{\mu_{q0} + (\mu_{h0} - \mu_{q0})h(\tau)}{[4a_q + 4(a_h - a_q)h(\tau)]^{2/3}} \frac{s^{2/3}}{\tau^2}. \quad (28)$$

Without the second and third terms on the right hand side, this equation describes the conservation of entropy. The second term is responsible for the entropy production due to nucleation. For the Maxwell construction, $p_h - p_q = 0$ as $T = T_c$ and this term vanishes. The third term leads to continuous entropy production due to dissipative effects. We discuss the solution of these equations in the next section.

4 Results and Discussion

By solving together Eq. (27) and Eq. (7), we have studied the plasma evolution and calculated $s\tau$ —the entropy production as a function of τ . The initial conditions are taken at $\tau = \tau_c$ as $T = T_c$, $h = 0$ and $\epsilon(\tau_c) = \epsilon_q(\tau_c)$. Now, $h(\tau)$ is calculated using Eq. (25) with step Δ in τ . With this value of h , Eq. (27) is solved for $\epsilon(\tau)$ [or $T(\tau)$]. Then Eq. (7) is evaluated by the trapezoidal rule, thereby yielding new value of h . Using new value of h , we solve again Eq. (27) to improve the value of ϵ . This is repeated till an accuracy of $\sim 10^{-5}$ is obtained. Then we proceed to the next step $\tau_c + 2\Delta$. We take $a_q = 37\pi^2/90$ and $a_h = 4.6\pi^2/90$. The value 4.6 is taken instead of 3 to account for ρ , ω and η mesons apart from pions. We chose $T_c = 160$ MeV in this work implying $B^{1/4} = 219$ MeV. For the hadronic matter $\eta_{h0} = 1.5$, $\zeta_{h0} = 1$ while for the quark matter $\zeta_{q0} = 0$ [7, 9, 10]. The coefficient η_{q0} has been varied from 2.5 to 20. The nucleation involves the surface free energy, correlation length, and the velocity of bubble growth, which have already been described in Sec. II. The volume of a fluid element is $A_\perp \tau \delta y$. As $\tau \delta y$ does not change with τ in the scaling hydrodynamics, $s\tau$ provides a measure of total entropy. So as to understand the role of supercooling and viscous heat generation, we have compared various scenarios; ideal hydrodynamics (IHD), IHD with supercooling, viscous hydrodynamics (VHD) and VHD with supercooling. We assumed that at initial time $\tau_i = 1$ fm/c, the temperature $T_i = 268$ MeV marked the beginning of the evolution. This corresponds to energy density $\epsilon = 8.51$ GeV/fm³ and entropy $s_i \tau_i = 40.65$ fm⁻². These are appropriate for the Relativistic Heavy Ion Collider (RHIC) energies. For further discussions, it is instructive to examine τ versus T plot. Figure 1 shows such a plot for VHD calculations with supercooling included. From an initial point (τ_i, T_i) , one moves on to (τ_c, T_c) where critical temperature is reached. However, the nucleation rate is zero at this point and system does not hadronize. The system keeps cooling to (τ_m, T_m) where

significant nucleation rate and hadronization is reached. The system cannot cool further as entropy cannot decrease according to the second law of thermodynamics. At this point, the hadron fraction is around 11-18 % for various cases. As the temperature increases towards T_c , the hadron fraction h increases and slowly approaches a value of one at the point (τ_h, T_c) . After this, temperature starts decreasing finally reaching the point (τ_f, T_f) at the freeze-out temperature T_f . This is the general character of $\tau - T$ curve. If one uses the Maxwell construction, one reaches directly from (τ_c, T_c) to (τ_h, T_c) without any change in the temperature.

In Figs. 2 and 3, we show the results for T , h and $s\tau$ as a function of τ for $\eta_{q0}=2.5$ and 5. For IHD, the critical temperature T_c is reached earlier than VHD. Similarly with supercooling included, T_m is approached earlier in IHD than VHD. The supercooling leads to an abrupt change in entropy starting from τ_m onwards, i.e., in the reheating region. We find in our calculations that $s\tau$ does not increase much beyond $\tau \approx 20$ fm/c. Therefore, $s_h\tau_h$ represents reasonably well the total entropy produced. As s_h is evaluated at T_c for all cases, the total entropy is also a measure of the life time of the system upto the hadronization stage. Figure 4 shows $s_h\tau_h$ as a function of η_{q0} . With VHD, entropy production increases with viscosity. Supercooling leads almost to a constant shift of the curve for the VHD. In Fig. 5, the excess entropy production due to supercooling is shown as a function of viscosity. For all values of η_{q0} , excess entropy does not change much. It is almost constant for the VHD calculation. We also found that the values of $\eta_{q0} \ll 1$ do not lead to significant hadronization and the system continues to cool in the QGP phase. Sufficient value of η_{q0} is required for the transition to the hadron phase. The bubble growth velocity coefficient ($3c$ in our calculation) also plays an important role in the transition as setting it to zero also blocks the transition. The viscosity plays very crucial role in the transition through nucleation. However, once viscosity enters into picture, the viscous hydrodynamics is to be used for consistency. In the presence of viscosity, the initial energy density estimates from the experimental rapidity density distribution would be lower than the ones calculated with the Bjorken formula. Though supercooling does not lead to significant increase in entropy production, it is very much needed. For the description of the phenomena in relativistic nuclear collisions, the viscous hydrodynamics with supercooling is a more appropriate framework than the ideal hydrodynamics with or without supercooling. Further theoretical work is needed to compute the viscosity with narrower bounds for the analysis of the experimental data.

5 Conclusions

We have studied the entropy production both in ideal and viscous scaling hydrodynamics with and without supercooling. Excess entropy produced due to supercooling in viscous

hydrodynamics, weakly depends on viscosity of the plasma phase. Though, in general entropy produced due to supercooling is much less than that due to viscous heat generation, the phenomenon of supercooling provides an important physical mechanism for the quark-hadron transition to occur. The viscous hydrodynamics with supercooling also leads to an increase in the lifetime of the plasma.

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- FIG. 1. T/T_c as a function of proper time τ in viscous hydrodynamics with supercooling.
- FIG. 2. The T/T_c , $h(\tau)$ and $s\tau$ as a function of τ in IHD (dotted), IHD with supercooling (short dashed), VHD (long dashed) and VHD with supercooling (solid) at $\eta_{q0}=2.5$.
- FIG. 3. Same as Fig. 2, but with $\eta_{q0}=5.0$.
- FIG. 4. $s_h\tau_h$ versus η_{q0} in IHD, IHD with supercooling, VHD and VHD with supercooling.
- FIG. 5. Excess entropy produced due to supercooling versus η_{q0} both with IHD and VHD.

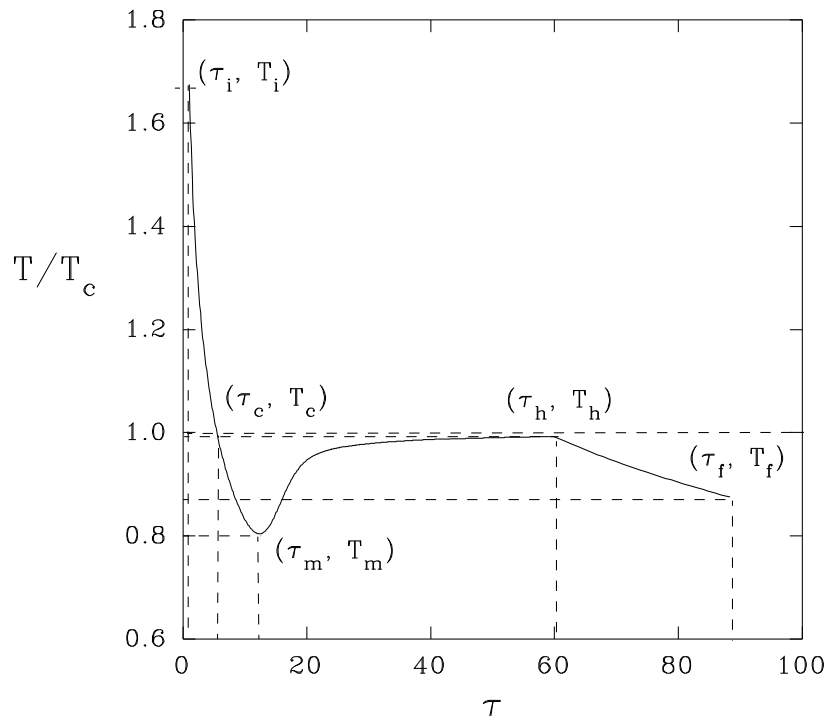


Fig. 1

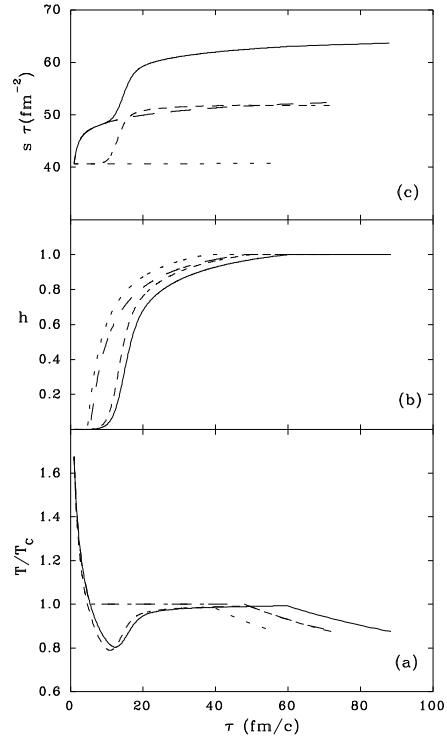


Fig. 2

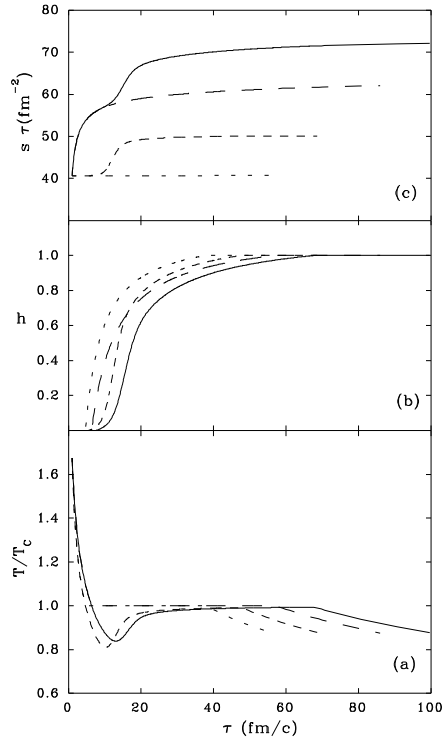


Fig. 3

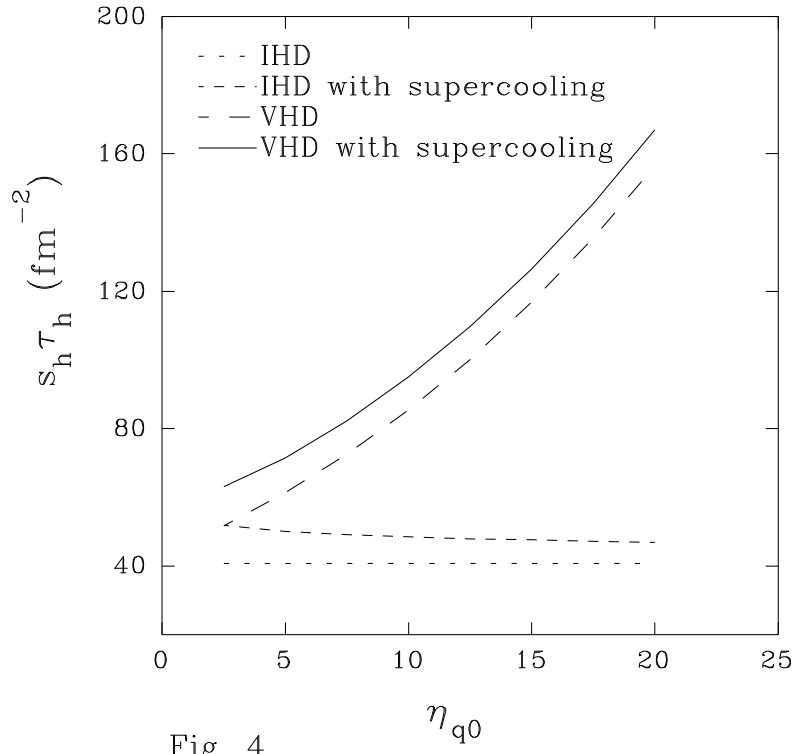


Fig. 4

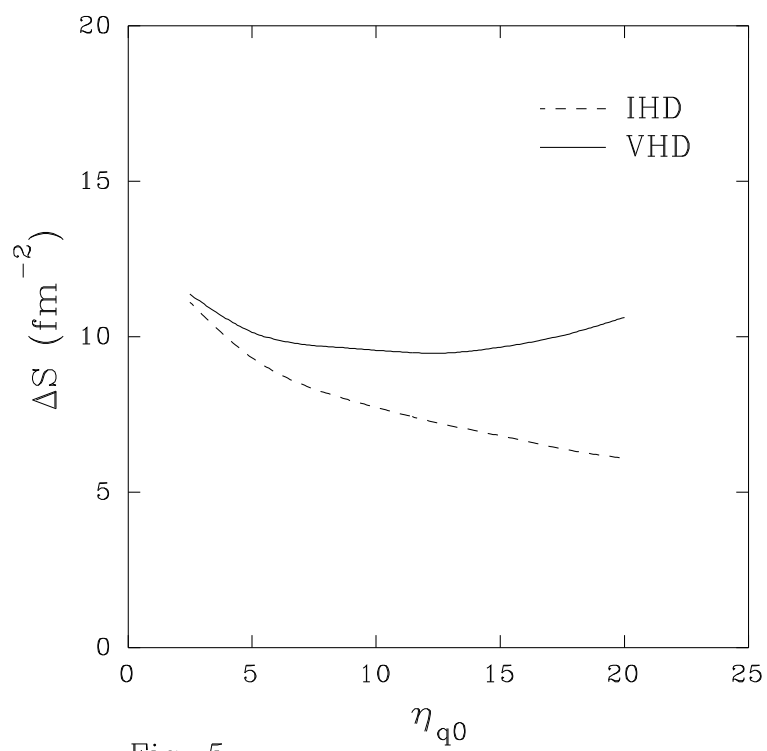


Fig. 5